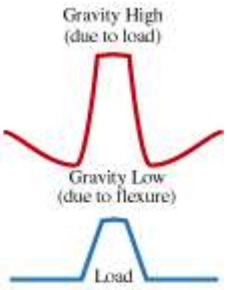


"4.1.BMP"

CALCULATE THE GRAVITY ANOMALY DUE TO THE LOAD OF HAWAII AND ESTIMATE THE ELASTIC THICKNESS OF THE OCEANIC LITHOSPHERE, T_e.

This Mathcad file calculates the gravity associated with the flexure of the oceanic crust by the load of the Hawaiian-Emperor seamount chain and compares it to the observed free-air gravity anomaly in the region of Oahu. The elastic thickness and load and infill density that best explains the observed gravity anomaly can be determined from the comparison.

Define input parameters

Define elastic thickness, T_e, average gravity, g, Poisson's ratio, v, Youngs Modulus, E, and the Universal Gravitational constant, G.

$Te := 25km$

$g := 9.81 m \cdot sec^{-2}$

$v := 0.25$

$E := 10^{11} Pa$

$G := 6.67 \cdot 10^{-11} \cdot newton \cdot m^2 \cdot kg^{-2}$

Define npts, the number of sample points along the gravity and topography profile. npts must be a

power of 2 (for the Fourier Transform).

$npts := 128$

Define density of the load (i.e. the seafloor topography),water and mantle

$\rho\_crust := 2800 kg \cdot m^{-3}$

$\rho\_water := 1030 kg \cdot m^{-3}$

$\rho\_mantle := 3330 kg \cdot m^{-3}$

Define normal (ie unflexed) thickness of oceanic crust

$ocean\_thick := 6km$

$$D := \frac{E \cdot (Te)^3}{12 \cdot \left(1 - v^2\right)}$$

Read in the observed topography and free-air gravity anomaly data

$M := \mathrm{READPRN}(\text{"haw\_topo.prn"})$

$N := \mathrm{READPRN}(\text{"haw\_grv.prn"})$

$icountt := \mathrm{rows}(M) - 1$

$profile := \left(M_{icountt,0} - M_{0,0}\right) km$

$j := 0 .. \mathrm{rows}(M) - 1$

$a_j := M_{j,0} \, km$

$b_j := M_{j,1} \, m$

$j := 0 .. \mathrm{rows}(N) - 1$

$c_j := N_{j,0} \, km$

$$d_j := \frac{N_{j,1}}{10^5} m \cdot sec^{-2}$$

Interpolate the observed data and take out a mean (for the Fourier transform)

$$dx := \frac{profile}{npts - 1}$$

$$XKINT := \frac{2 \cdot \pi}{npts \cdot dx}$$

$$XINT := \frac{profile}{npts - 1}$$

$i := 0 .. npts - 1$

$x_i := a_0 + (i \cdot XINT)$

$obs\_topo_i := \mathrm{linterp}\left(a, b, x_i\right)$

$obs\_grv_i := \mathrm{linterp}\left(c, d, x_i\right)$

$mean\_depth := -\mathrm{mean}(obs\_topo)$

$i := 0 .. npts - 1$

$obs\_topo_i := obs\_topo_i + mean\_depth$

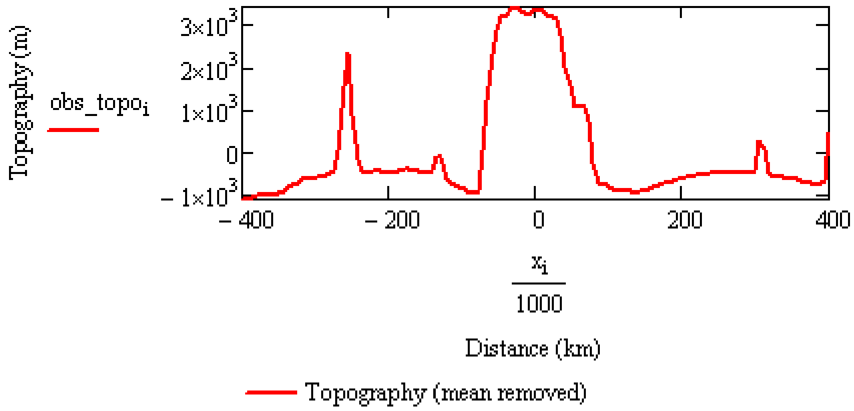
$mean\_gravity := \mathrm{mean}(obs\_grv)$

$i := 0 .. npts - 1$

$obs\_grv_i := obs\_grv_i - mean\_gravity$

$mean\_depth = 3.825 \times 10^3 \cdot m$

Display observed topography data



Calculate the flexural response function, Phi, and the gravitational admittance based on a flexure model, Zflexure

$c := \mathrm{fft}(obs\_topo)$

$k := 0, 1 .. \frac{npts}{2}$

$$Phi_k := \left[\frac{D\cdot(k\cdot XKINT)^4}{g\cdot(\rho\_mantle - \rho\_crust)} + 1\right]^{-1}$$

(See Eq. 5.18)

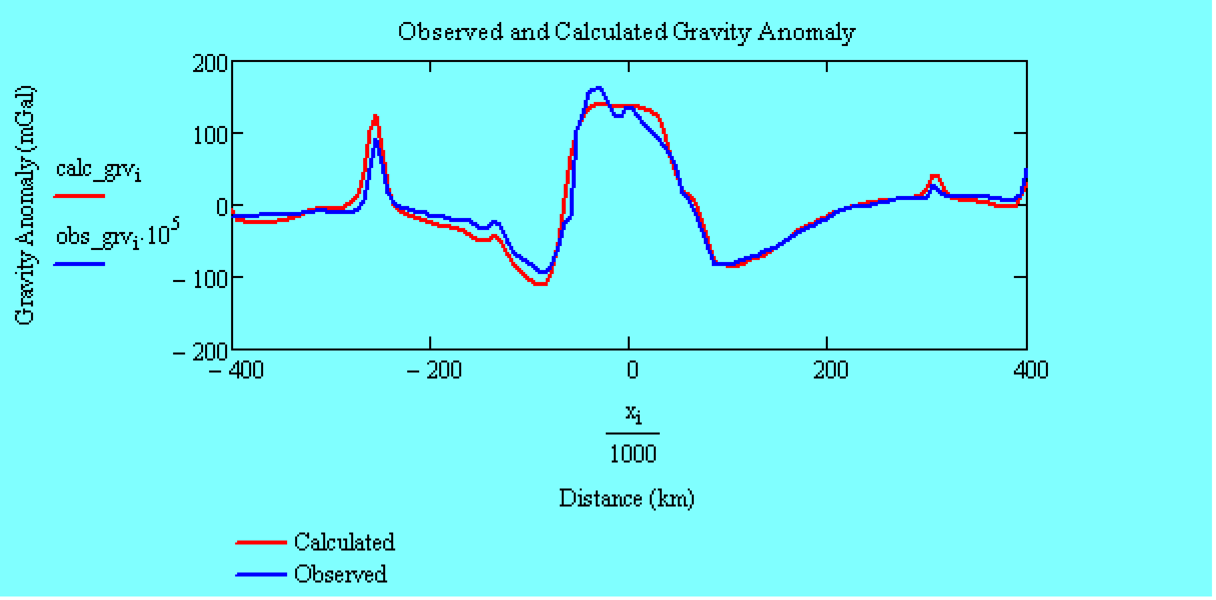
$$Zflexure_k := 2\cdot\pi\cdot G\cdot 10^5\cdot(\rho\_crust - \rho\_water)\cdot e^{-(k\cdot XKINT\cdot mean\_depth)}\cdot\left[1 - \left(Phi_k\cdot e^{-XKINT\cdot ocean\_thick}\right)\right]$$

(See Eq. 5.19)

$factor_k := Zflexure_k\cdot c_k$

$calc\_grv := \mathrm{ifft}(factor)$

Display results



Note: the "best fit" value of Te is one that explains both the amplitude and wavelength of the observed gravity anomaly.

Calculate correlation coefficient, r, between observed and calculated gravity anomalies

$r := \mathrm{corr}(obs\_grv, calc\_grv)$
$r = 0.972$

Calculate variance and RMS error between observed and calculated gravity anomalies

$$diff := obs\_grv - \frac{calc\_grv}{10^5}$$

Convert to m.sec^{-2}

$variance := \mathrm{var}(diff)$
$rms := \sqrt{variance}$
$rms = 1.491 \times 10^{-4} \cdot m \cdot s^{-2}$

Multiply by 10^5 to get mGal

This gives the RMS difference (and correlation coefficient) between observed and calculated gravity anomalies based on the values of elastic thickness and densities assumed above.